Gauge Threshold Corrections and Field Redefinitions

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Abstract

We review the argument for field redefinitions arising from threshold corrections to heterotic string gauge couplings, and the relation between the linear and the chiral multiplet. In the type IIB case we argue that the necessity for moduli mixing at one loop order has not been clearly established, since this is based on extending the background field expansion way beyond its regime of validity. We also resolve some issues related to the form of non-perturbative terms resulting from gaugino condensation. This enables us to estimate the effective cutoff in the field theory by evaluating the non-perturbative superpotential by two different methods, and find that it is around the Kaluza-Klein scale, as one might have expected on general grounds of self-consistency.

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1 Introduction

Background field methods in string theory (see for example [1][1, 2, 3] are different from what one encounters in say, the quantum mechanical study of the behavior of an atom in an external magnetic field. In the latter case the background magnetic field is truly external to the system under study. In field theory applied say to condensed matter physics or atomic physics, where one is studying not the theory of the entire universe but some system within it, the concept of an external background makes sense. However when one studies theories such as the standard model coupled to gravity, which purports to be an effective theory of the entire universe, strictly speaking there is no meaning to the concept of an external background.

Of course the standard model is usually formulated in a particular metric background - namely the flat one. Here the reasoning is that for small standard model field energy densities, the Einstein equations are solved by the Minkowski metric. In principle it can be studied in a different metric background for example a cosmological background (FRW, deSitter etc.). However this smooth gravitational background field assumption is certainly expected to break down close to the Planck scale ($M_P \equiv 1/\sqrt{8\pi G_{Newton}}$). At such high energies one expects a significant contribution from virtual quantum gravity processes (such as the creation and annihilation of blackholes, wormholes etc.) and the entire framework will break down.

String theory on the other hand is supposed to be an UV completion of field theory (or at least of a class of field theories hopefully including the standard model). The theory is not supposed to have any free parameters and is defined purely in terms of a fundamental dimensional constant - the string scale $l_{string} \equiv \sqrt{2\pi\alpha'} = 1/M_{string}$. If string theory were four dimensional, M_{string} would essentially be the same as M_P. However in all tractable string theoretic constructions, there is an internal six-dimensional space with a volume which is typically large compared to l_{string}^6, i.e. $\mathrm{Vol} = \mathcal{V} l_{string}^6$ often with $\mathcal{V} \gg 1$. In this case there is a significant difference between the two scales and $M_S \simeq M_P/\sqrt{\mathcal{V}} \ll M_P$. There is also an additional scale, the Kaluza-Klein scale $M_{KK} = M_{string}/\mathcal{V}^{1/6} = M_P/\mathcal{V}^{2/3}$. For large $\mathcal{V}$ we thus have a hierarchy of scales $M_{KK} \ll M_{string} \ll M_P$. Four dimensional field theory is strictly valid only below M_{KK}. Above this scale the theory is essentially ten dimensional but remains a field theory. However above the scale M_{string}, the theory *cannot* be described by point like field theoretic degrees of freedom. The field theoretic description necessarily breaks down.

Consider first the case of strings propagating in a general metric background. In this case the world sheet theory is formulated as a generalized two dimensional sigma model, and one derives consistency conditions (beta function equations) for the propagation of strings, in an expansion in the squared string length scale - the so-called α' expansion. For energy scales that are well below the string scale, this is a valid expansion and one can get useful information about the low energy limit of string theory in this way. However this expansion obviously breaks down at the string scale. In fact this is highlighted by the fact that the derivative expansion (as with generic higher derivative theories) has ghosts. These however appear at the string scale and are merely a sign that the theory has been pushed beyond its regime of validity. Thus any argument that is made about the interaction vertices of the theory - that is derived from the α' expansion - is invalid when the momentum flowing through those vertices is greater than the string scale. At such energies the low energy point field theory needs to be replaced by string (field?) theory.

The same is true for open string background calculations. Here one turns on a gauge field

strength (magnetic or electric) background that is slowly varying (if not constant), to derive a low energy effective action in an α' expansion. Much important work has been done by using this technique. However for the most part this work has been used only to get an effective field theory valid below the string scale. In particular it does not make sense to consider the behavior of a term like $\frac{1}{g^2(\mu^2)}\mathrm{tr}(F_{u\nu}F^{u\nu})$ when the momentum flowing through this operator is greater than the string scale. The representation in terms of such a local operator simply breaks down at these energies. In particular this means that any argument which purports to have a low (around say few TeV) string scale and gauge coupling unification at some much higher scale (such as the standard GUT scale of $10^{16}GeV$), cannot possibly make sense.

In this work we will address first the question of string threshold corrections to gauge couplings by first reviewing the literature. The issue came up with Kaplunovsky's calculation of these effects in the context of the Heterotic string [4]. The question that arose was how to account for moduli dependent corrections that could not be written as harmonic functions of the moduli as would be required in the usual chiral field formulation of the effective supergravity coming from string theory. The resolution in terms of the linear and chiral multiplet duality, was discussed in [5][6] (BGG) and is reviewed in section two, where we also point out that the argument fails when the modulus in question has a non-linear superpotential term in the relevant chiral superfield. In the next section we review the arguments of Kaplunovsky and Louis which compared their field theoretic formula for the gauge coupling function with the corresponding string theory calculation. It should be stressed that this only involved momentum scales below the string scale (as we will make clear below). By contrast the work of references [7, 8, 9], as well as the earlier work of [10, 11, 12] in connection with low scale strings, is essentially based on using the effective field theory and background string theory approaches well above the string scale. The argument for the necessity of certain field redefinitions that result from this comparison are then very sensitive to exactly where the cutoffs are located, and can be changed by appropriately choosing the cutoffs at scales below the string scale.

2 General framework

2.1 Linear -chiral duality with $\partial_S W = 0$.

As mentioned earlier the linear multiplet - chiral multiple duality has been discussed for example in [5] and [6](BGG). We essentially follow the discussion of BGG except that the Kaehler supergravity framework is replaced by the standard (minimal) one. We begin with the following action (with $\kappa = M_P^{-1} = 1$, $d^8z = d^4xd^4\theta$, $d^6z = d^4xd^2\theta$) for chiral superfields Φ (having superpotential W and Kaehler potential K) coupled to supergravity and gauge fields with prepotential V and gauge field strength $\mathcal{W}$.

$$\begin{aligned} \mathcal{A} &= -3\int d^8z\mathbf{E}\exp[-\frac{1}{3}K(\Phi,\bar{\Phi};V)] \\ &\quad + \left(\int d^8z\frac{\mathbf{E}}{2R}[W(\Phi)+\frac{1}{4}f(\Phi)\mathcal{W}\mathcal{W}]+h.c.\right). \end{aligned} \tag{1}$$

Note that the gauge coupling function f is in general a *holomorphic* gauge invariant function of the chiral superfields Φ. However threshold effects in string theory appeared to give non-holomorphic

moduli dependent corrections to the gauge coupling function. The resolution lay in the introduction of the linear multiplet formulation of the gauge coupling function. The key observation here is that string theory moduli and the dilaton naturally arise in the string theory context as (components of) linear multiplets. This is because axionic partners of the scalar moduli are in fact second rank tensor fields. Thus for instance the axio-dilaton S, which is often identified in 4D as a chiral scalar, has its origins in a multiplet containing an antisymmetric second rank tensor $b_{\mu\nu}$, and thus naturally belongs to a linear multiplet.

Let us first focus on this case where in the usual formulation the gauge coupling function is given by $f = kS$ where S is the dilaton chiral superfield i.e. $\bar{\nabla}^{\dot{\alpha}}S = 0$. Let $\{\phi\}$ denote all the other chiral superfileds in the theory. We take (for the moment) the superpotential W to be independent of S as is the case in perturbative string theory (except in IIB where it can be linear in S in the presence of internal fluxes). Let U be an unconstrained real superfield and modify the above action to the following form (for simplicity we ignore chiral fields which are charged under the gauge group),

$$\begin{aligned}\mathcal{A} &= -3\int d^8z\mathbf{E}\exp[-\frac{1}{3}K(\phi,\bar{\phi},U)](F(\phi,\bar{\phi},U)+U(S+\bar{S})) \\ &\quad + \left(\int d^8z\frac{\mathbf{E}}{2R}[W(\phi)+\frac{1}{4}kS\mathcal{W}\mathcal{W}]+h.c.\right). \end{aligned} \tag{2}$$

Here a trace over the gauge group is implicit in the gauge kinetic term, F is a real function of the chiral fields ϕ and the real field U, which will be determined by a normalization condition below. Note that we have also modified the Kaehler potential to include dependence on U. Now we may eliminate the chiral superfield S in favor of the real superfield U, by using the equation of motion coming from taking the δS variation of this action to get; [1]

$$-3(-\frac{1}{4}\bar{\nabla}^2+2R)(Ue^{-K/3})+\frac{k}{4}\mathcal{W}^2=0. \tag{3}$$

Note that this equation and its conjugate are now effectively constraints on the initially unconstrained superfield U. In the absence of the gauge field kinetic term and K this is essentially the linear superfield constraint. Here we have a modified linear superfield.

Substituting (3) into (2) we get the (modified) linear multiplet formulation of the above action

$$\begin{aligned}\mathcal{A}_{LMF} &= -3\int d^8z\mathbf{E}e^{-K(\phi,\bar{\phi},U)/3}F(\phi,\bar{\phi},U) \\ &\quad +(\int d^8z\frac{\mathbf{E}}{2R}W(\phi)+h.c.). \end{aligned} \tag{4}$$

Note that there is no explicit gauge kinetic term in this form of the action. It is however implicit because U satisfies the constraint (3). To see this we first rewrite this as an equation for $-\frac{1}{4}\bar{\nabla}^2U$ to get (keeping only terms which contain $\mathcal{W}^2$)

$$-\frac{1}{4}\bar{\nabla}^2U=\frac{e^{K/3}}{1-\frac{1}{3}UK_U}\frac{k}{3\times 4}\mathcal{W}^2+\ldots.$$

[1] In taking a variation w.r.t. a chiral field we need to set $\delta S=(-\frac{1}{4}\bar{\nabla}^2+2R)\delta\Sigma$ where Σ is an unconstrained superfield.

Then we have from the first line of (4), using $\int d^8z\mathbf{E}\bar{\nabla}^{\dot{\alpha}}v_{\dot{\alpha}}=0$ and the above result,

$$
\begin{aligned}
-3\int d^8z\frac{\mathbf{E}}{2R}(-\frac{1}{4}\bar{\nabla}^2+2R)\left(e^{-K/3}F(\phi,\bar{\phi},U)\right) &= \\
-3\int d^8z\frac{\mathbf{E}}{2R}e^{-K/3}(-\frac{K_U}{3}F+F_U)(-\frac{1}{4}\bar{\nabla}^2U)+\ldots &= -\int d^8z\frac{\mathbf{E}}{2R}\frac{k}{4}\Gamma(U,\phi,\bar{\phi})\mathcal{W}\mathcal{W}+\ldots,
\end{aligned}
$$

so that the effective gauge coupling function in the linear multiplet formulation is (the lowest component of)

$$-\Gamma=-\frac{F_U-FK_U/3}{1-UK_U/3}, \tag{5}$$

in agreement with BGG.

Now let us see what we would get in the chiral field formulation of the gauge coupling function. Varying the action (2) with respect to U we get

$$(S+\bar{S})(1-\frac{1}{3}UK_U)=\frac{1}{3}FK_U-F_U. \tag{6}$$

This equation determines $U=U(S+\bar{S},\phi,\bar{\phi})$. In the chiral field formulation we need to have in addition the normalization condition

$$F(\phi,\bar{\phi},U)+U(S+\bar{S})=1. \tag{7}$$

This is just the condition that in the chiral formulation the pre-factor of $e^{-K/3}$ should be unity. Substituting into (2) gives the chiral field formulation with the standard chiral gauge coupling function i.e. (1).

Eliminating $(S+\bar{S})$ between (6) and (7) we get a differential equation for F,

$$-U^2\frac{d}{dU}(U^{-1}F)=1-\frac{1}{3}UK_U.$$

This has the solution

$$F=1+\frac{U}{3}\int^U\frac{dU'}{U'}K_{U'}+V(\phi,\bar{\phi})U. \tag{8}$$

A simple example that is relevant to string theory, is obtained by putting

$$K=\hat{K}(\phi,\bar{\phi})+\alpha\ln U. \tag{9}$$

In this case $F=1-\frac{\alpha}{3}+VU$ and $\frac{\alpha}{3}U^{-1}=S+\bar{S}+V(\phi,\bar{\phi})$. Then in the chiral multiplet formulation we have the action (1) with $f=kS$ and

$$K=\hat{K}(\phi,\bar{\phi})-\alpha\ln(S+\bar{S}+V(\phi,\bar{\phi})). \tag{10}$$

In the string theory case where S is the four dimensional dilaton chiral superfield $\alpha=1$ and V is a one-loop effect.

On the other hand we have in the linear multiplet formulation the action (4) with Kaehler potential (9), the gauge coupling (see (34)) given by (putting $\alpha=1$)

$$\Gamma=-\frac{1}{3U}+V(\phi,\bar{\phi}). \tag{11}$$

The moral of this story is that if one wants to accommodate a non-harmonic gauge coupling function (as in the above relation) one must choose the LM formulation. On the other hand this is equivalent to taking the chiral multiplet dual field S as the gauge coupling function in the CM formulation but with the non-harmonic part V included in the Kaehler potential for S.

The above can be generalized in straightforward way to the case when there are several superfileds that undergo a duality transformation (see for example [6]).

2.2 $\partial_S W \neq 0$

The discussion in the previous subsection remains valid if the superpotential dependence on S is no more than linear. This is the case for instance in type IIB string theory, provided the pre-factors of the non-perturbative terms responsible for breaking the no-scale structure and stabilizing the Kaehler moduli are independent of S (or at least no more than linear in S). Thus under these usual assumptions one has

$$W = A(\phi) + B(\phi)S, \tag{12}$$

where as in the last subsection ϕ stands for all the other chiral scalar superfields. In this case the only relevant change is in equation (3), which is replaced by

$$-3(-\frac{1}{4}\bar{\nabla}^2 + 2R)(Ue^{-K/3}) + \frac{k}{4}\mathcal{W}^2 + B(\phi) = 0.$$

Equation (4) will remain the same with $W \to A(\phi)$ but on evaluating the components of the first term we would need to replace $k\mathcal{W}^2 \to k\mathcal{W}^2 + B(\phi)$. The main results of the previous subsection remain unchanged.

However when the superpotential is non-linearly dependent on S these results cannot be obtained. The reason is that the δS equation no longer gives a constraint on the unconstrained superfield U. Instead it becomes an equation which expresses the chiral superfield S in terms of a chiral projection of an unconstrained superfield. Defining $\mathbf{P} = (-\frac{1}{4}\bar{\nabla}^2 + 2R)$ equation (3) is replaced by ($\partial_S W \equiv W_S$),

$$-3\mathbf{P}(Ue^{-K/3}) + \frac{k}{4}\mathcal{W}^2 + W_S = 0. \tag{13}$$

Thus we need to replace S in equation (2) by

$$S = W_S^{-1}(3\mathbf{P}(Ue^{-K/3}) - \frac{k}{4}\mathcal{W}^2). \tag{14}$$

Substituting (13) into (2) (with $W(\phi) \to W(\phi) + \Delta W(S)$ we have for the chiral superspace terms

$$\int d^8 z \frac{\mathbf{E}}{2R}[W(\phi) + (\Delta W(S) - S\partial_S \Delta W)] + h.c. \tag{15}$$

where S is now given as a function of U by (14). Clearly gauge kinetic terms are hidden in this expression, but the coupling functions are essentially power series in $\mathbf{P}(Ue^{-K/3})$ which can always be rewritten as a chiral projection of some redefined unconstrained real field Ω i.e. as $\mathbf{P}\Omega$. For

instance in the simplest case where $\Delta W = S^2/2$ the expression (15) becomes

$$\int d^8z \frac{\mathbf{E}}{2R}[W(\phi) - \frac{1}{2}(3\mathbf{P}(Ue^{-K/3}) - \frac{k}{4}\mathcal{W}^2)^2] + h.c. \sim$$
$$\int d^8z \frac{\mathbf{E}}{2R}[W(\phi) - (3\mathbf{P}(Ue^{-K/3})\frac{k}{4}\mathcal{W}^2] \quad + \quad h.c.$$

so that the gauge coupling function is $3\mathbf{P}(Ue^{-K/3})$. As can be easily seen this is the case whatever the functional form ΔW takes, since this relates to the coefficient of the leading term in the expansion in $\mathcal{W}^2$. This is obviously of the same form as the original chiral field representation in terms of S for the gauge coupling function, and is very different from the linear multiplet formulation where it was precisely the constraint on U that enabled us to write a non-holomorphic gauge coupling.

3 String theory considerations

The KL formula [13] for the gauge coupling constant for a (simple or $U(1)$ gauge group G_a) is

$$\begin{aligned}
\frac{1}{g_a^2(\Phi, \bar{\Phi}; \mu^2)} \quad &= \quad \Re f_a(\Phi) + \frac{b_a}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} + \frac{c_a}{16\pi^2} K(\Phi, \bar{\Phi}) \\
&+ \frac{T(G_a)}{8\pi^2} \ln \frac{1}{g^2(\Phi, \bar{\Phi}; \mu^2)} - \sum_r \frac{T_a(r)}{8\pi^2} \ln \det Z_{(r)}(\Phi, \bar{\Phi}; g^2(\mu^2)). \qquad (16)
\end{aligned}$$

Here $T_a(r) = \mathrm{Tr}_{(r)} T_a^2$, $T(G_a) = T_a(r = \text{adjoint})$,

$$b_a = \sum_r n_r T_a(r) - 3T(G_a), \qquad (17)$$

$$c_a = \sum_r n_r T_a(r) - T(G_a). \qquad (18)$$

Φ stands for a set of neutral fields that will be identified with string theory moduli (M) and axio-dilaton (S). The total Kaehler potential has been expanded in powers of the charged matter chiral fields Q which also defines the metric $Z_{(r)}$ of their kinetic terms in the representation r of the gauge group, i.e.

$$K(\Phi, \bar{\Phi}; Q, \bar{Q}) = \kappa^{-2} K(\Phi, \bar{\Phi}) + \sum_r Z_{(r)\bar{I}J}(\Phi, \bar{\Phi}) \bar{Q}^{\bar{I}}_{(r)} e^{2V} Q^J_{(r)} + \ldots . \qquad (19)$$

The above formula was derived within an effective supergravity context on the understanding that at some scale above the effective cutoff (Λ), this field theory would have to be replaced by its ultra-violet completion. Here the latter will be assumed to be string theory.

Several comments about the formula (16) are in order here. The first term on the RHS is the (real part of the) holomorphic gauge coupling of the theory defined in (1). Its functional form may be read off from the relevant string theory whose low-energy effective action is being studied. It will also include the holomorphic corrections coming from integrating out massive

string states. The second term is the usual field theory running from the cutoff scale down to the scale μ. The fourth term comes from the rescaling anomaly that comes when the gauge field prepotential is redefined so as to get (from the standard superspace form in (1)) to the canonically normalized form of the gauge/gaugino field kinetic terms [14]. The fifth term comes from the Konishi anomaly whose origin is in the the rescaling necessary to get the canonical kinetic terms for the chiral scalar/fermion fields. These all occur already in global SUSY and together constitute the (integrated form of) the NSVZ beta function equation. In particular they have nothing to do with rescaling the (super) metric and only involve rescaling the fields Q and V. By contrast the third term comes from the anomaly that occurs when performing the Weyl transformations (on the supermetric) that are necessary to go to the Einstein-Kaehler frame starting from the superspace frame of (1).

3.1 Heterotic case

The expression (16) is expected to be valid to all orders in perturbation theory (at least in some renormalization scheme) but we will focus only on the one loop result. The holomorphic gauge coupling is (for the Heterotic string and for the gauge theory on D3 branes in type IIB)

$$f_a(S,M) = k_a S + \frac{1}{16\pi^2} f_a^{(1)}(M). \tag{20}$$

The first term is the classical (universal) gauge coupling and the second is the holomorphic one-loop correction. Of course f receives no further corrections. The other terms are already explicitly of at least one-loop order so that K, g_a^2, Z can be replaced by their classical values. Thus we put

$$\begin{aligned}
K(\Phi,\bar{\Phi}) &= -\ln(S+\bar{S}) + \hat{K}(M,\bar{M}) + O(\frac{1}{16\pi^2\Re S}), \\
Z_{\bar{I}J}(\Phi,\bar{\Phi}) &= Z^{(0)}_{\bar{I}J}(M,\bar{M}) + O(\frac{1}{16\pi^2\Re S}), \\
\frac{1}{g^2(\mu^2)} &= \Re S + O(\frac{1}{16\pi^2}).
\end{aligned}$$

It is important to note that in the heterotic case the classical 4 dimensional string coupling is defined as

$$\frac{1}{g^2_{string}} = \Re S = e^{-2\phi}\mathcal{V}$$

where $\mathcal{V}$ is the volume of the internal space in string units and e^{ϕ} is the 10 dimensional string coupling. Then we get from (16)

$$\begin{aligned}
\frac{1}{g_a^2(\Phi,\bar{\Phi};\mu^2)} &= k_a\Re S + \frac{b_a}{16\pi^2}(\ln\frac{\Lambda^2}{\mu^2} - \ln\Re S) \\
&\quad + \frac{1}{16\pi^2}[\Re f_a^{(1)}(M) + c_a\hat{K}(M,\bar{M}) - \sum_r 2T_a(r)\ln\det Z^{(0)}_{(r)}(M,\bar{M})].
\end{aligned} \tag{21}$$

This is to be compared with the string 1 loop calculation [15] [2]

$$\frac{1}{g_a^2(\Phi,\bar{\Phi};\mu^2)} \quad = \quad k_a\frac{1}{g_{string}^2(\Phi,\bar{\Phi})}+\frac{b_a}{16\pi^2}\ln\frac{M_{string}^2}{\mu^2}+\frac{\Delta_a(M,\bar{M})}{16\pi^2} \tag{22}$$

where

$$\frac{1}{g_{string}^2(\Phi,\bar{\Phi})}=\Re S+\frac{\Delta^{univ}(M,\bar{M})}{16\pi^2} \tag{23}$$

Now in the Heterotic string $M_{string}^2=M_P^2/\Re S$ up to S,M independent constants. So it appears that the S dependence of the two formulae (21)(22) agrees. It should be stressed here that although in [15] the cut-off Λ is identified with M_P this is merely a matter of convenience, and as the authors observed all that is required is that in the field theory expression the cutoff Λ should be chosen in an S,M independent manner.

As is well known the natural superspace variable that corresponds to the axio-dilaton of string theory is the linear multiplet (since the four dimensional axion is originally a two-form field). Suppose for the moment we ignore the (non-universal) second line (21) as well as the last term on the RHS in (22). As discussed in the previous section, at least as long as we do not include a superpotential term for the chiral superfield S in the chiral multiplet form (CMF) action[3], the linear multiplet form is equivalent to the chiral multiplet form. According to the discussion there, the gauge coupling function (at $\mu^2=M_{string}^2$) is to be interpreted initially as the expression Γ in (34). The model (9) (11) then corresponds to the LMF form with

$$\Gamma=-\frac{1}{3U}+V(M,\bar{M}).$$

Here U should correspond to the dilaton in the linear multiplet formulation which to zero loop order can be identified as $-(3\Re S)^{-1}$. The term V in (11) is identified with $\frac{\Delta^{univ}(M,\bar{M})}{16\pi^2}$. In the corresponding chiral multiplet formulation (in terms of S) the instruction implied by this is in effect to drop this term from the expression for the coupling function (thus identifying at the high scale $g_a^{-2}=k_a\Re S$) but include it as a one-loop correction to K (see(10)) with

$$K=-\ln(S+\bar{S}+\frac{\Delta^{univ}(M\bar{M})}{16\pi^2})+\hat{K}(M,\bar{M}). \tag{24}$$

Note that if one makes the above correction to $K$and then plugs that back into the CMF form of the action (1), then all that changes in the KL formula (16) is the explicit expression for the term $\frac{c_a}{16\pi^2}K(\Phi,\bar{\Phi})$ which now becomes $\frac{c_a}{16\pi^2}$ times the RHS of (24). However in (16) this is already a one-loop effect so this one-loop change in K is not going to affect the KL formula to one loop.

3.2 Type IIB case

Again we start with the 1-loop form of the KL formula (16) but now we write

$$K^{(0)}=-2\ln\mathcal{V}+\tilde{K}((S,\bar{S};z,\bar{z}). \tag{25}$$

[2]See also [16] where an additional universal contribution is identified.

[3]Actually as observed in subsection 2.2 one could have a linear term - but in the heterotic case there is no such term. The only possible dilaton superpotential comes form gaugino condensation and is a sum of exponentials in S.

We have separated the Kaehler moduli dependence (through the internal volume $\mathcal{V}$) from the complex structure (z) dependence. Note that here in contrast to the heterotic case the appropriate axio-dilaton field is

$$S = e^{-\phi} + ia$$

where e^{ϕ} is the 10 dimensional string coupling, and a is the four dimensional dual of the RR two form field (see for example [3]). As in the LVS models discussed in [17][18] it is assumed that the MSSM is located on D3 branes at a singularity (or D7 branes wrapping a collapsing cycle). For these local models one can write $Z^{(0)}_{\bar{I}J} = \frac{1}{\mathcal{V}^{2/3}}\delta_{\bar{I}J}$ to leading order in the large volume expansion. Then the one-loop coupling function becomes

$$\begin{aligned}\frac{1}{g_a^2(\Phi,\bar{\Phi};\mu^2)} &= \Re f_a(\Phi) + \frac{b_a}{16\pi^2}\ln\frac{\Lambda^2\mathcal{V}^{-2/3}}{\mu^2} + \frac{c_a}{16\pi^2}\tilde{K}((S,\bar{S};U,\bar{U}) \\ &\quad + \frac{T(G_a)}{8\pi^2}\ln\Re f_a(\Phi). \end{aligned} \tag{26}$$

In [7] the cutoff Λ is taken to be M_P. As we discussed before there is no physical significance in the actual value of this cut off as long as it is taken to be independent of the moduli. It is simply the upper limit from which one expects the RG evolution to start, and if the field theory is a low energy effective theory whose UV completion is string theory then this scale must necessarily be less than the string scale. As discussed in [15], for the purpose of comparing the moduli dependence of this formula to that coming from string theory, it does not matter at what point Λ is chosen (as long as it is above all low energy thresholds) and indeed without loss of generality it can be chosen to be M_P. This obviously does not imply any statement about the validity of the field theory up to this point and all the conclusion of that paper would still hold even if the arbitrary cutoff Λ is kept.

The conclusion of [7] (see also [8]) however depends crucially on the choice $\Lambda = M_P$. With this the argument of the log in the second term of the (26) is

$$\frac{\Lambda^2\mathcal{V}^{-2/3}}{\mu^2} \to \frac{M_P^2\mathcal{V}^{-2/3}}{\mu^2} \simeq \frac{M_{string}^2 R^2}{\mu^2}, \tag{27}$$

where we've used (the approximate) formula $M_{string}^2 \simeq M_P^2/\mathcal{V}$ and put $R \equiv \mathcal{V}^{1/6}$ the size of the internal space in string units. From this it is concluded that the effective unification scale (for large volume compactifications) can be far above the string scale[4]. As the authors themselves point out in a footnote it is not clear what the "operational" meaning of this is. Clearly it cannot have any, for if instead of the above choice of $\Lambda = M_P$ we took a fixed value of $\Lambda \lesssim M_{string}$ (as one should)[5], then the corresponding unification scale would be below the string scale! However in [8] the authors go on to derive physical conclusions based on having a cutoff that is larger than the string scale.

These come from the authors' comparison of (26) (after replacing $\Lambda \to M_P$) with a string theory calculation. Based on a background expansion of the sort that we discussed in the introduction,

[4]Note that to have unification one not only needs $f_a \propto k_a$ but also that the third and fourth terms on the RHS of (26) should be negligible.

[5]The RHS of this inequality is of course moduli dependent. So what this requires is that one has to first decide on the values over which the volume is allowed to range and then fix Λ below the lowest allowed value.

the following equation is obtained in [8],

$$\frac{1}{g_a^2(\mu^2)} = \frac{1}{g^2}|_0 + \beta_a \ln \frac{M_{string}^2}{\mu^2} + \beta_a^{\mathcal{N}=2} \ln \frac{M_X^2}{M_{string}^2}. \tag{28}$$

Here $\beta_a = \beta_a^{\mathcal{N}=1} + \beta_a^{\mathcal{N}=2}$ corresponding to the contributions to the beta function from $\mathcal{N} = 1, 2$ states and $M_X^2 = R^2 M_{string}^2$. The last two terms on the RHS of the above equation are obtained from cutting off two UV divergent integrals of the form

$$\beta_a^{\mathcal{N}=1} \int^{\mu^{-2}} \frac{dt}{t}, \ \beta^{\mathcal{N}=2} \int^{\mu^{-2}} \frac{dt}{t}. \tag{29}$$

The upper limit of these integrals is set by the infra-red RG scale μ of the previous discussion. On the other hand the UV cut-off is identified in the first case with M_{string}^{-2} while in the second case it is identified with the (even smaller!) winding scale M_X^{-2}. As we've pointed out in this note, one should not use the background field calculation right at the string scale. Thus this kind of argument cannot be used to make any statement about the modular dependence, since all that one needs to have agreement with the field theory calculation is to choose the UV cutoff in the above integrals to be the same, and to take the value

$$\Lambda_{string}^2 = \frac{\Lambda^2}{\mathcal{V}^{2/3}} < \frac{M_{string}^2}{\mathcal{V}^{2/3}}. \tag{30}$$

This cutoff is (for $\mathcal{V} > 1$) obviously well within the regime of validity of the background field method. In other words there is really no new information in this calculation. It just gives us the translation to string language of the choice of cutoff in the field theory. All that is required for consistency is that both are well under the string scale!

The point is not so much to argue for the above cutoff in the string theory calculation, as to show that these calculations are intrinsically ambiguous[6].

4 Non-perturbative superpotentials and the UV cutoff

In order to discuss this it is convenient to rewrite the manifestly supersymmetric superspace supergravity action i.e. (1), in a manifestly super-Weyl invariant form;

$$\begin{aligned} \mathcal{A} &= -3 \int d^8 z \mathbf{E} C \bar{C} \exp[-\frac{1}{3} K(\Phi, \bar{\Phi}; \mathcal{V})] + \\ & \left(\int d^6 z \mathcal{E} [C^3 W(\Phi) + \frac{1}{4} f_a(\Phi) \mathcal{W}^a \mathcal{W}^a] + h.c. \right). \end{aligned} \tag{31}$$

Note that we have also generalized the action slightly in order to incorporate more than one gauge group and have used the chiral representation in which $\int d^2\bar{\theta} \mathbf{E}/2R = \mathcal{E}$ the chiral density. In this form the action has an additional manifest symmetry under the transformations

$$\begin{aligned} \mathbf{E} \to \mathbf{e}^{\mathbf{2}(\tau+\bar{\tau})} \mathbf{E}, &\qquad \mathcal{E} \to e^{6\tau} \mathcal{E} + \ldots, \ C \to e^{-2\tau} C, \\ \nabla_\alpha \to e^{(\tau - 2\bar{\tau})} (\nabla_\alpha - \ldots), &\qquad V \to V, \\ \Phi \to \Phi, &\qquad \mathcal{W}_\alpha \to e^{-3\tau} W_\alpha. \end{aligned} \tag{32}$$

[6]For related comments see the published version of [19].

The chiral (auxiliary) field C is introduced so as to make the Weyl invariance of the theory manifest. It is important to note that the superpotential occurs in this action with a factor of C^3. Now the above (chiral) Weyl transformations are anomalous in the quantum theory, and the preservation of this local Weyl invariance requires that the gauge coupling function is changed as follows: [13]

$$f_a(\Phi) \to f_a(\Phi) - \frac{3c_a}{8\pi^2} \ln C, \tag{33}$$

where c_a was given in (18). There is however additional C dependence coming from a field redefinition anomaly which occurs on demanding canonical normalization for the matter kinetic terms (see eqn.(16) of [20]). Thus the actual C dependence changes from the second term of (33) to $-\frac{b_a}{8\pi^2} \ln C$. Now suppose that the gauge group becomes strongly coupled and develops a mass gap below some scale. Then below that scale there is an effective theory that is obtained by integrating out the gauge theory degrees of freedom. This gives an effective action Γ defined schematically by

$$e^{-\Gamma(\Phi,\bar{\Phi},C,\bar{C})} = \int d(gauge) \exp\left\{-\frac{1}{4}\int [f_a(\Phi) - \frac{b_a}{8\pi^2}\ln C]\mathcal{W}^a\mathcal{W}^a + h.c.\right\}. \tag{34}$$

Since SUSY should not be broken by this procedure, we expect Γ [7] to have the general form of a superspace action and in particular should develop a superpotential. Given the general argument (based on Weyl invariance above) that any superpotential should come with a factor C^3, we see that the corresponding term in Γ will be (a superspace integral of)

$$C^3 W_{NP} = C^3 A_a \exp\left(-3\frac{8\pi^2}{b_a} f_a(\Phi)\right). \tag{35}$$

Here A_a is an $O(1)$ pre-factor (from dimensional analysis this would mean $A = O(M_P^3)$), which may depend on the moduli due to threshold corrections. If there is more than one condensing gauge group, there will obviously be a sum of such terms. This is essentially the Veneziano-Yankielowicz [21] argument generalized to SUGRA (see [22]). There is also of course a contribution to the Kaehler potential, but this is less important since unlike the superpotential, the Kaehler potential is perturbatively corrected. The total superpotential is then given as

$$W = W_c(\Phi) + W_{NP} = W_c(\Phi) + Ae^{-3\frac{8\pi^2}{b_a} f_a(\Phi)}, \tag{36}$$

where the first term on the RHS is the classical superpotential. It should be stressed that this argument is not at all dependent on the Weyl compensator formalism. If we had worked with $C = 1$ (as in Wess and Bagger [23]), then the form of the NP term (35) is what is required to get the right Kaehler transformation of the superpotential as a result of the Kaehler anomaly (which is now related to the Weyl anomaly) and renormalization of the matter kinetic term.

On the other hand there is in the literature an alternative form for W_{NP} based on an RG evolution argument. First one observes that the IR scale Λ_a (at which the theory becomes strongly coupled) is related to the UV scale by

$$\Lambda_a^3 = \Lambda^3 e^{-3\frac{2\pi\tau}{b_a}} = \Lambda^3 \mathcal{V} e^{-3\frac{8\pi^2}{b_a}\Re f_a(\Phi)}. \tag{37}$$

[7]The crucial assumption here is the quasi-locality of Γ which enables us to define its derivative expansion and then focus on its two derivative action which should be be of the standard supergravity form.

Here $\tau \equiv 4\pi/g_a^2$ where g_a is the coupling at the UV scale Λ and for the last equality we've used eqn. (26) and ignored $O(1)$ corrections to the prefactor.

In the global SUSY literature one often sees the evaluation of the superpotential as $|W_{NP}| = \Lambda_a^3$. However in SUGRA as pointed out in [9] this should be replaced by including a factor which arises from transforming to the Einstein frame. This comes from the C^3 factor in (35), after gauge fixing $\ln C + \ln \bar{C} = K/3$, the value that is needed to go to Einstein frame. Thus one should identify

$$e^{K/2}|W_{NP}| =< \mathcal{W}^a \mathcal{W}^a >= \Lambda_a^3 \tag{38}$$

From this after using $K \sim -2\ln \mathcal{V}$ and (37) we have

$$|W_{NP}| = \Lambda^3 \mathcal{V}^2 e^{-3\frac{8\pi^2}{b_a}\Re f_a(\Phi)}. \tag{39}$$

Comparing with (the second term of) (36) with $A \sim M_P^3$ we see that the cutoff Λ may be estimated to be

$$\Lambda \sim \frac{M_P}{\mathcal{V}^{2/3}} \sim \frac{M_{string}}{\mathcal{V}^{1/6}} = M_{KK}$$

In other words the cutoff should be at the KK scale and is safely below the string scale for large volume. This is in sharp contrast to the argument of [9].

5 Conclusions

We have argued that in the case of type IIB with the MSSM on D3 branes at a singularity, there is no need for moduli mixing unlike in the case of the heterotic string. In the latter case there is a universal one-loop correction coming from the string calculation, which can be reinterpreted in the chiral superfield formulation as a correction to the Kaehler potential. In the case of non-universal corrections to the small cycle of the LVS constructions, we have argued that there is no necessity for such redefinition. We have also shown, using two different arguments for the non-perturbative superpotential, that the effective cutoff is around the KK scale, in agreement with low energy effective action expectations.

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